1.5 Notes and Examples

Name: Block: Seat:

Infinite Limits

We conclude Chapter 1 with a distinction among the limits that show unbounded behavior. You may have heard me describe this a Type II limit, where a non-zero number is divided by zero. You have seen already that this causes vertical asymptotes.

Strategy: How to Find $\lim\limits_{x \to a} f(x)$

Try to evaluate $f(a)$ (i.e., replace x with a in the expression). You will get one of 3 things:

Type I. You got a number, c: You're done!

Type II. You got $\dfrac{\textit{not } 0}{0}$: a **vertical asymptote**: one of three possible conclusions:

1.
2.
3.

Type III. You got $\dfrac{0}{0}$ - See Section 1.3: Resort to Algebra tricks! (Factor and reduce, rationalize with a conjugate, trig identities, theorems like $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1$, $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{x} = 0$, etc.

N.B.: Just because you see $\lim f(x) = \infty$, or $\lim f(x) = -\infty$ it **does not** mean that the limit exists! On the contrary, it tells you **how** the limit fails to exist.

1. Let $f(x) = \dfrac{3}{x-2}$.

x	1.5	1.9	1.99	1.999	2	2.001	2.01	2.1	2.5
$f(x)$									

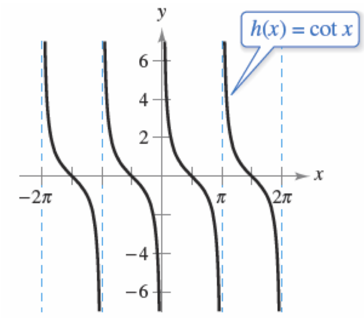
(a) $f(2) =$

(b) $\lim\limits_{x \to 2^-} f(x) =$_______ Meaning the $f(x)$ ______________ as x approaches 2 from the negative side.

(c) $\lim\limits_{x \to 2^+} f(x) =$ _______ Meaning the $f(x)$ ______________ as x approaches 2 from the negative side.

2. (a) $\lim_{x \to 0} \cot x$

   (b) $\lim_{x \to \pi} \cot x$



Function with vertical asymptotes

Properties of Infinite Limits:

If L and c are real numbers (i.e. $\in \mathbb{R}$), and f and g are functions where $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$, then

1. Sum Rule: $\lim_{x \to c} (f(x) + g(x)) =$
2. Difference Rule: $\lim_{x \to c} (f(x) - g(x)) =$
3. Product Rule (if $L > 0$): $\lim_{x \to c} (f(x) \cdot g(x)) =$
4. Product Rule (if $L < 0$): $\lim_{x \to c} (f(x) \cdot g(x)) =$
5. Quotient Rule (if $L \neq 0$): $\lim_{x \to c} \left(\frac{f(x)}{g(x)} \right) =$

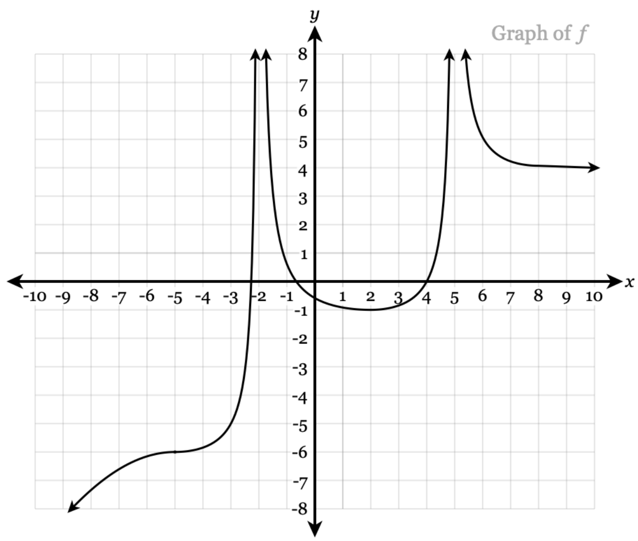
3. (a) $\lim_{x \to 0} 1 + \frac{1}{x^2} =$

   (b) $\lim_{x \to 1^-} \frac{x^2 + 1}{\cot \pi x} =$

   (c) $\lim_{x \to 0^+} 3 \cot x =$

   (d) $\lim_{x \to 0^-} \left(x^2 + \frac{1}{x} \right) =$

What about horizontal asymptotes?

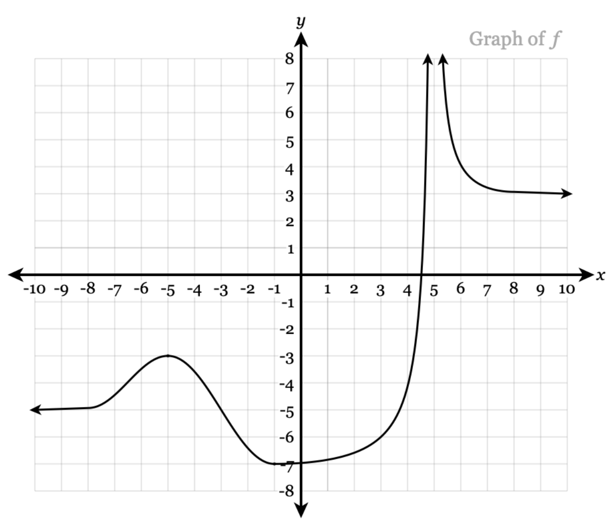


4. (a) $\lim_{x\to\infty} f(x) =$

   (b) $\lim_{x\to-\infty} f(x) =$

   (c) $\lim_{x\to-2} f(x) =$

   (d) $\lim_{x\to5} f(x) =$



5. (a) $\lim_{x\to\infty} f(x) =$

   (b) $\lim_{x\to-\infty} f(x) =$

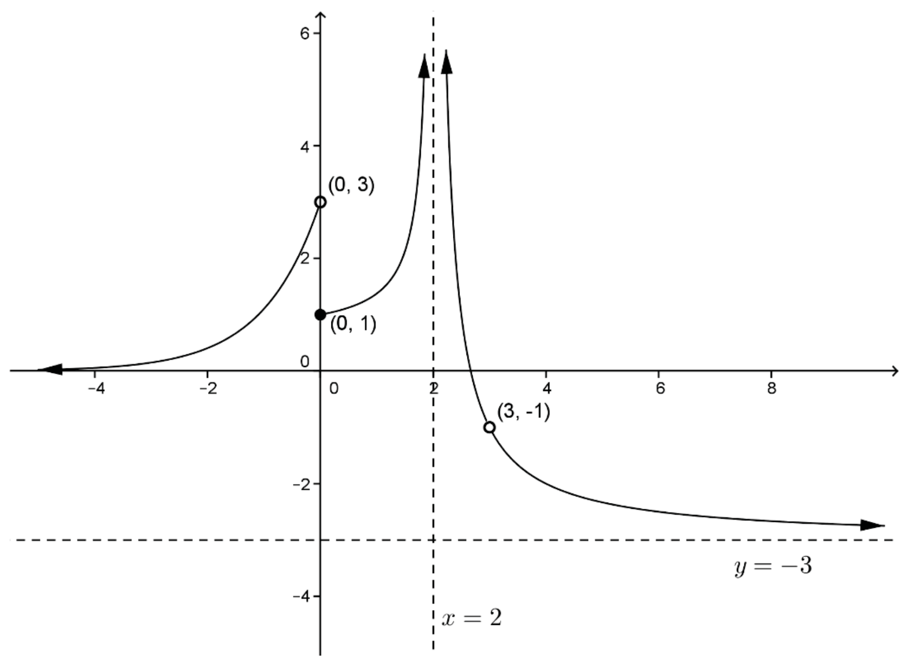
   (c) $\lim_{x\to5} f(x) =$

6. If $\lim_{x \to 3^-} g(x) = +\infty$ and $\lim_{x \to 3^+} g(x) = -\infty$, does this imply a vertical or horizontal asymptote? What else can you conclude?

7. If $\lim_{x \to -1^-} h(x) = 4$ and $\lim_{x \to -1^+} h(x) = -\infty$, does this imply a vertical or horizontal asymptote? What else can you conclude?

8. If $\lim_{t \to \infty} v(t) = 2$, does this imply a vertical or horizontal asymptote?

9. There are 11 limit statements you can make from the graph of f below. Can you find them all?



1. ____________________ 5. ____________________ 9. ____________________

2. ____________________ 6. ____________________ 10. ____________________

3. ____________________ 7. ____________________ 11. ____________________

4. ____________________ 8. ____________________